This implies that g_k has the pole of order $\overline{\delta}_k$ on $E_\sigma^{(\sigma)}$. On the other hand, by Lemma 1, g_{k+1} has the pole of order $q_k\overline{\delta}_k$ on $E_\sigma^{(\sigma)}$. Hence, $E_\sigma^{(\sigma)}$ is neither the zero nor the pole of $\Phi = \dfrac{g_{k+1}}{g_k^{q_k}}$. Further, Φ is holomorphic in a neighborhood of Q and $\Phi(Q) = 0$. Therefore, Φ is not constant on $E_\sigma^{(\sigma)}$.
Now, set $\psi = g_{k+1} - g_k^{q_k}$. Then,

$$\frac{\psi}{g_k^{q_k}} = \Phi - 1$$

is also a non-constant function on $E_\sigma^{(\sigma)}$. Therefore, ψ has also the pole of order $q_k\overline{\delta}_k$ on $E_\sigma^{(\sigma)}$. On the other hand, since

$$\deg_y(\psi) < n_{k+1} = n_k q_k,\ n_k = \deg_y(g_k),$$

by the division of ψ by $g_k^{q_k-1}$, we get

$$\psi = c_1 g_k^{q_k-1} + \psi_1$$

with $\deg_y(c_1) < n_k$, $\deg_y(\psi_1) < n_k(q_k-1)$. Dividing ψ_{i-1} by $g_k^{q_k-i}$ successively for $i = 2, \cdots, q_k - 1$, we get

$$\psi_{i-1} = c_i g_k^{q_k-i} + \psi_i,$$

where $\deg_y(c_1) < n_k$, $\deg_y(\psi_i) < n_k(q_k - i)$. Thus, setting $c_{q_k} = \psi_{q_k-1}$, we get

$$\psi = \sum_{i=1}^{q_k} c_i g_k^{q_k-i}.$$

Here, we have

$$\deg_y(c_i) < n_k = n_{k-1}q_{k-1},\ n_{k-1} = \deg_y(g_{k-1}).$$

In the same way, dividing c_i and its rests by $g_{k-1}^{q_{k-1}-1}$, $g_{k-1}^{q_{k-1}-2}$, $\cdots$, g_{k-1} successively, we get

$$c_i = \sum_{j=1}^{q_{k-1}} c_{ij} g_{k-1}^{q_{k-1}-j}$$

with $\deg_y(c_{ij}) < n_{k-1}$. Thus, we have

$$\psi = \sum_{i=1}^{q_k} \sum_{j=1}^{q_{k-1}} c_{ij} g_k^{q_k-i} g_{k-1}^{q_{k-1}-j}.$$